Network Flow Models

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Red Hot Research at The Commons
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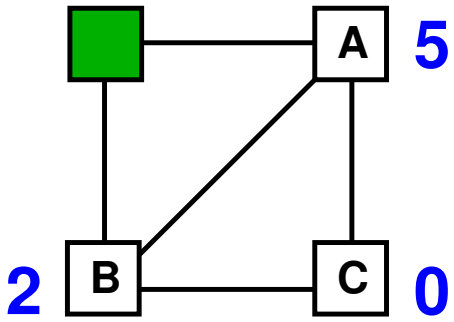
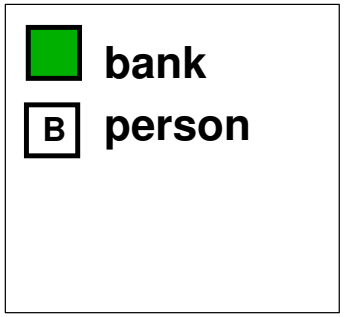
A Network Flow Model: The "Dollar Game"

- Underlying model: Network of people linked by friendships (think Facebook!)
- Math jargon: people are **vertices**, friendships are **edges**
- Each person has a pile of dollars
- If you have at least as many dollars as friends, go on a spending spree — "fire" a dollar to each friend
- Special vertex: **bank / government** (which rarely goes on a spending spree)

The Dollar Game





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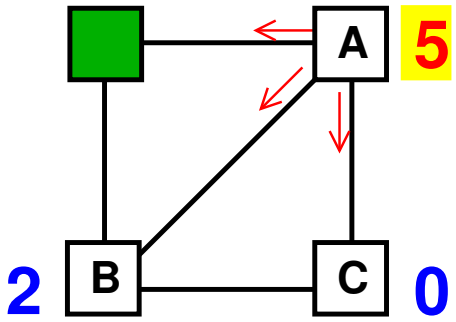

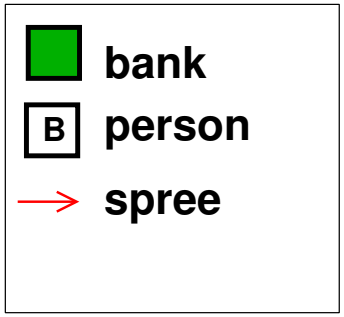

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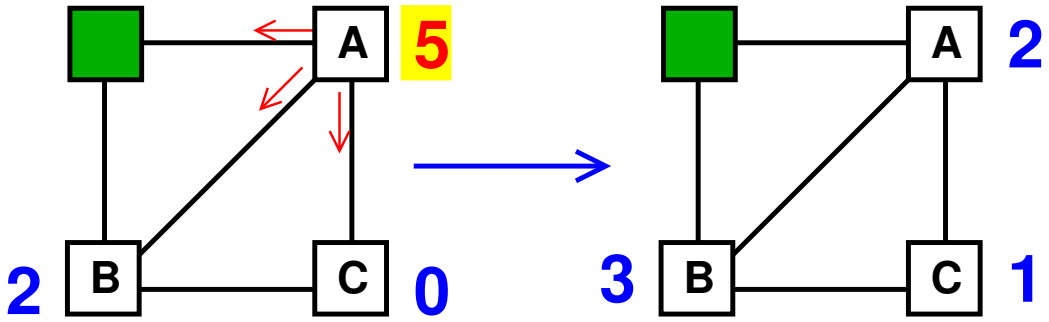

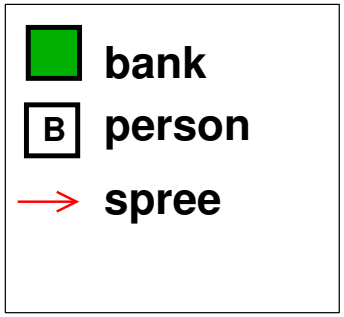

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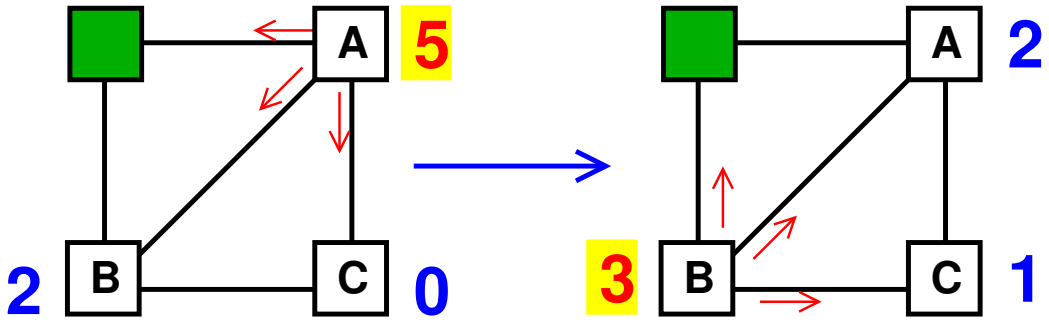

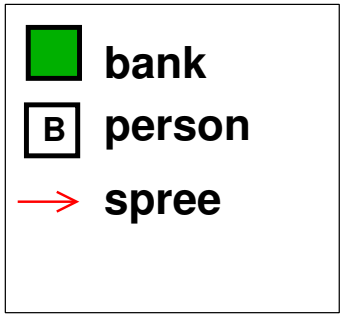

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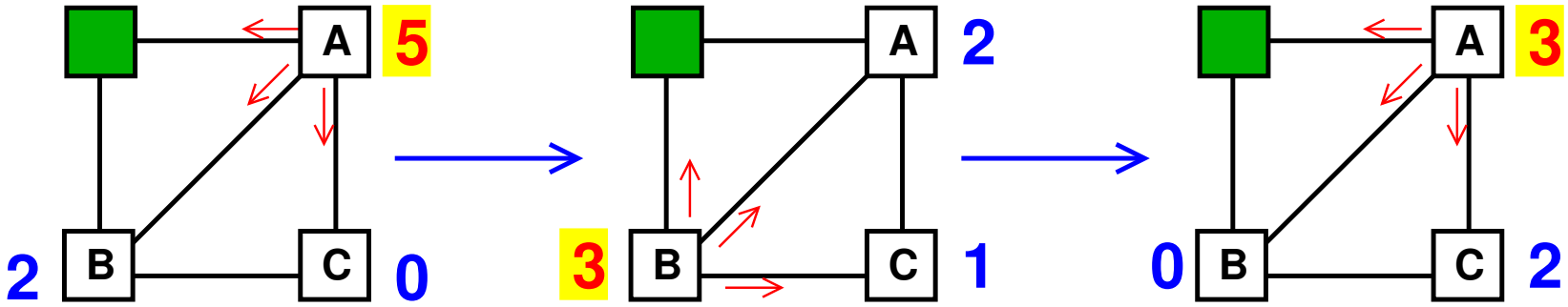

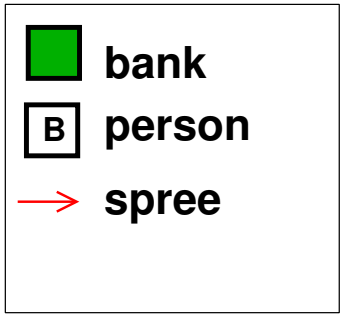

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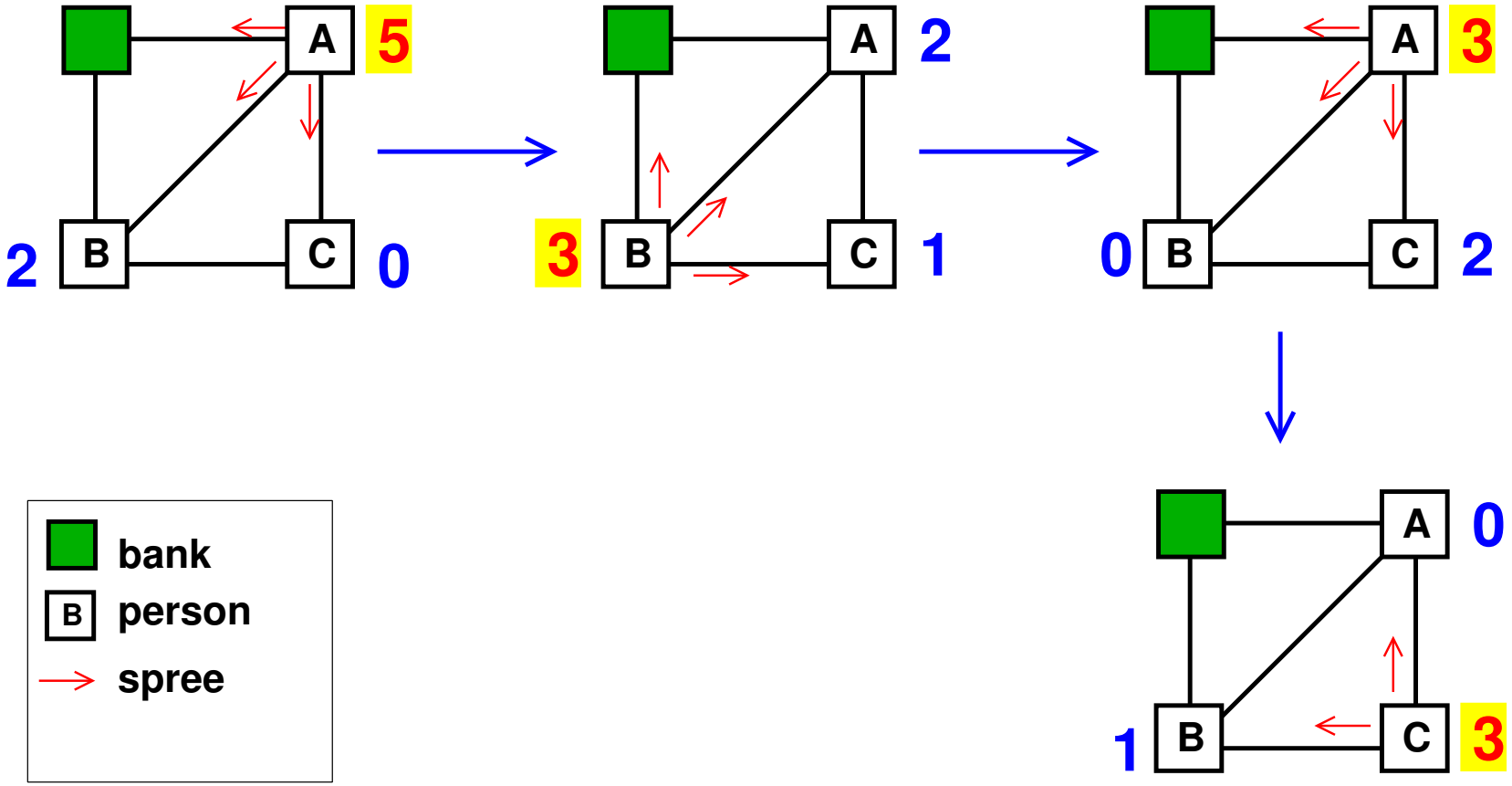

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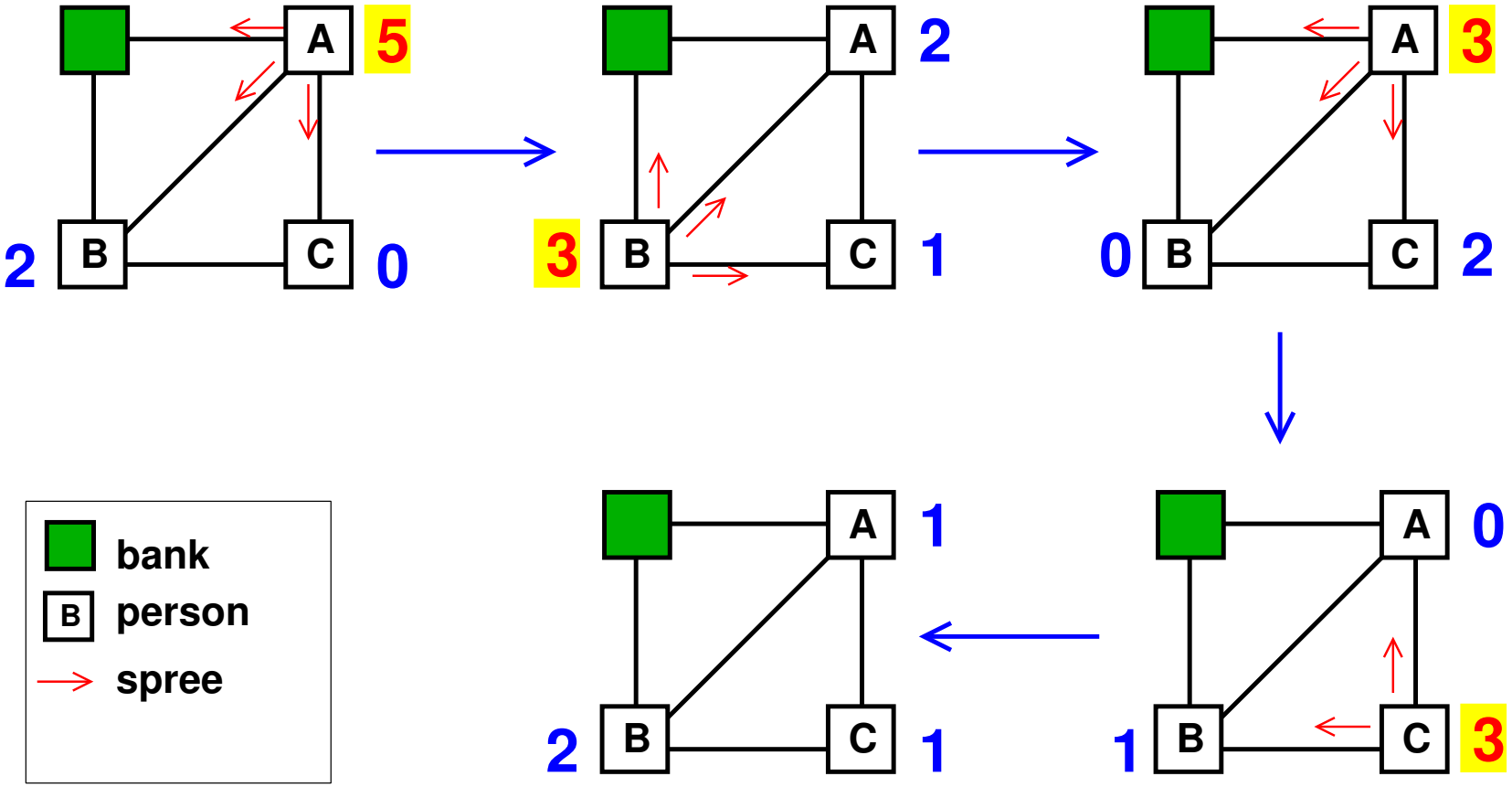

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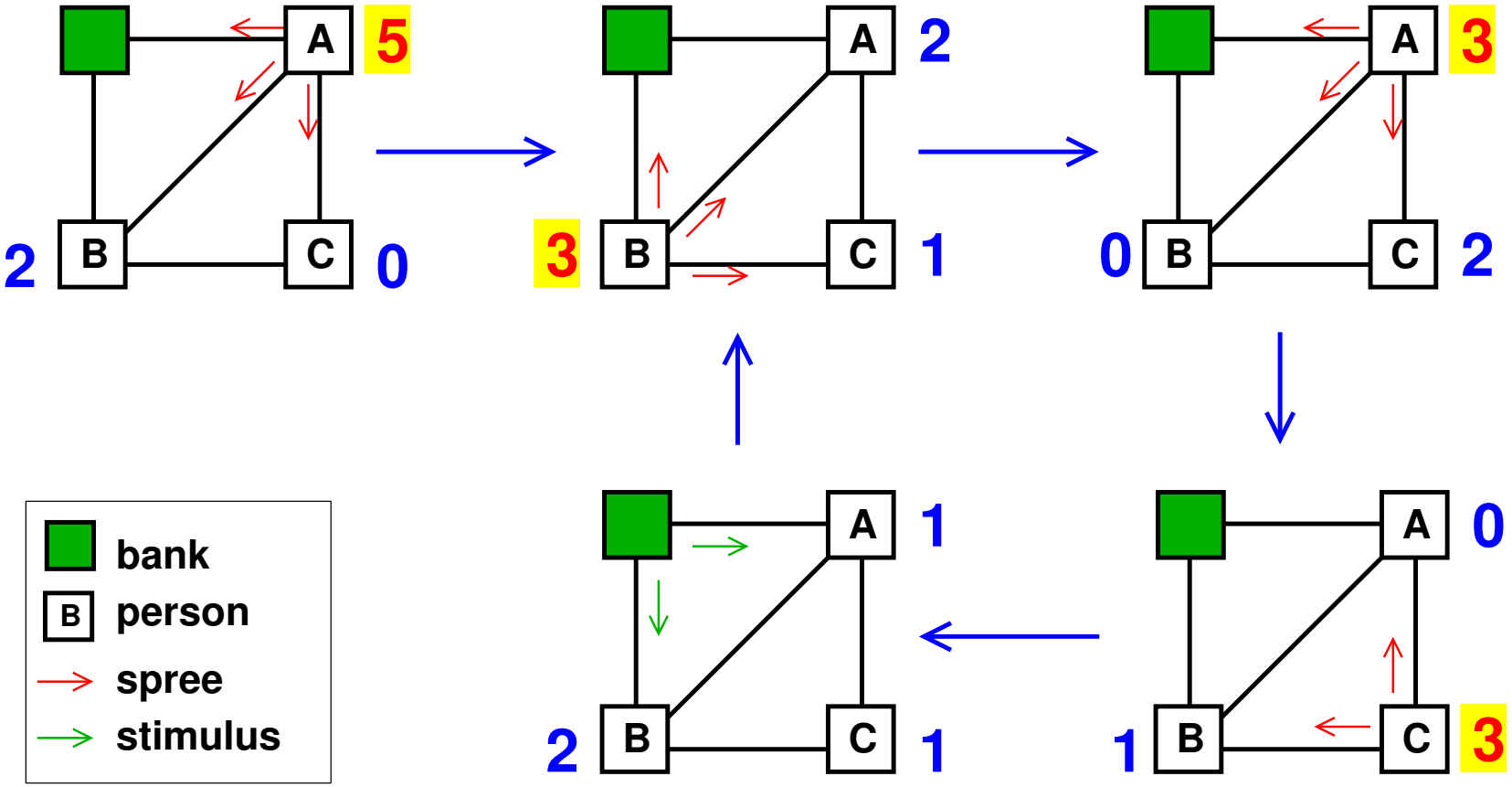

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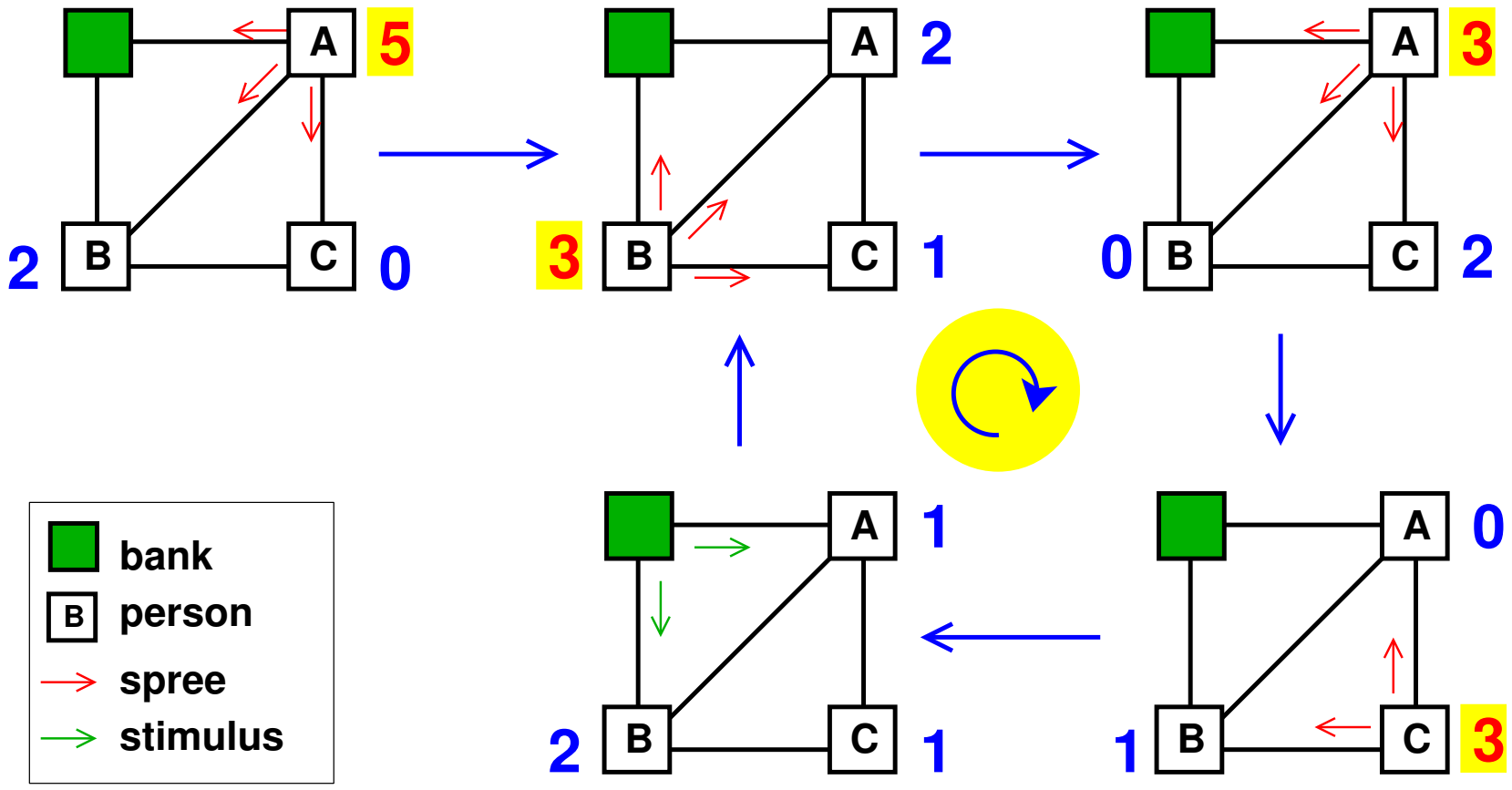

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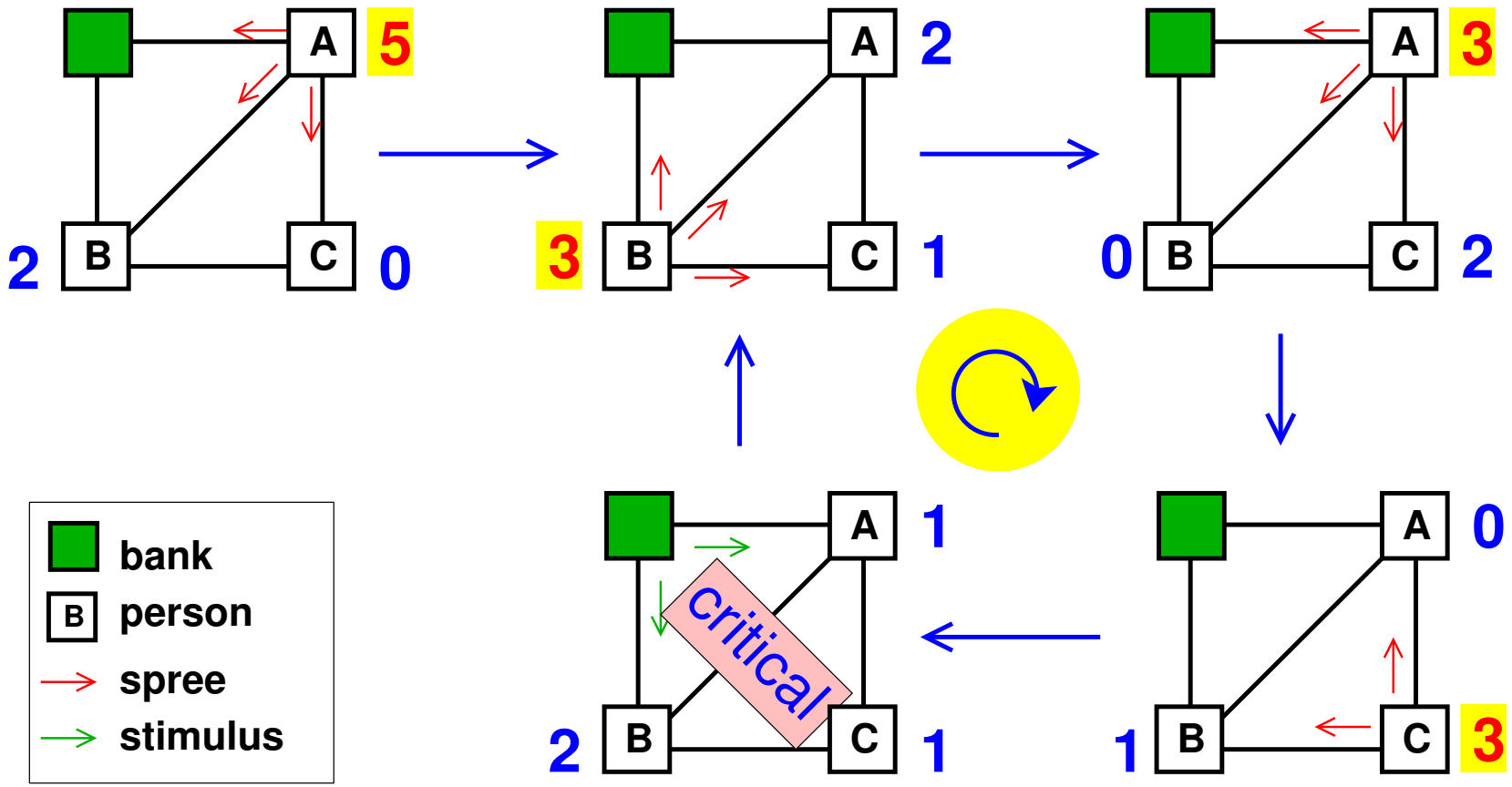


Why Study This Model?

- Statistical physics ("sandpile model")
  - How does instability spread throughout a system?
  - Modelling "ripple effects"
- Theoretical mathematics ("chip-firing")
  - Motivations in graph theory (study of general networks)
  - Number of critical configs = complexity of network
  - Geometry — singularities moving around curves

Flow in Higher Dimensions

Thought:

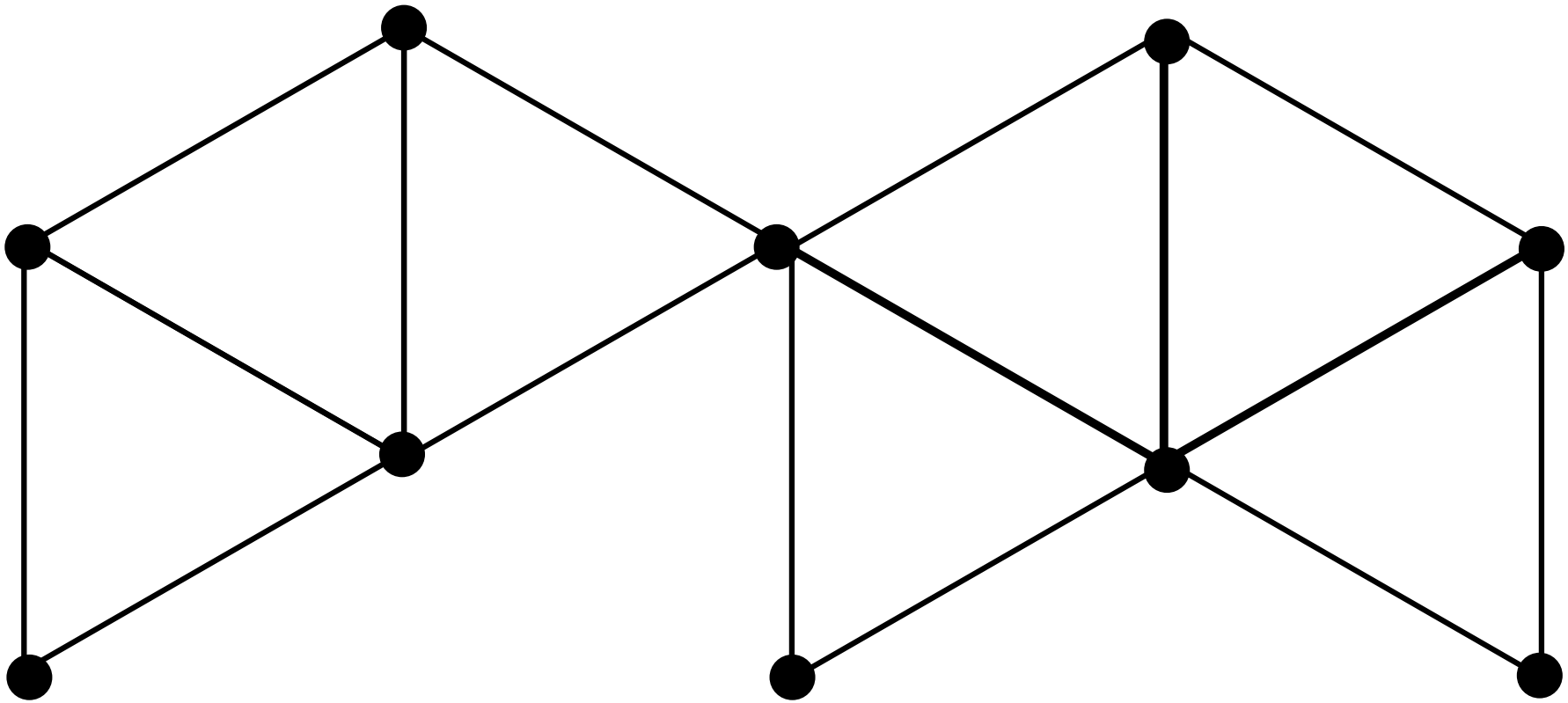
Dollars flow between 0-dimensional objects (vertices) along 1-dimensional pathways (edges).

Research Problem:

Build a **more general model** of flow between $(n-1)$-dimensional "objects" along n-dimensional "paths".

(The dollar game would be just the $n = 1$ case.)

Redirecting 1-D Flow Along 2-D Paths



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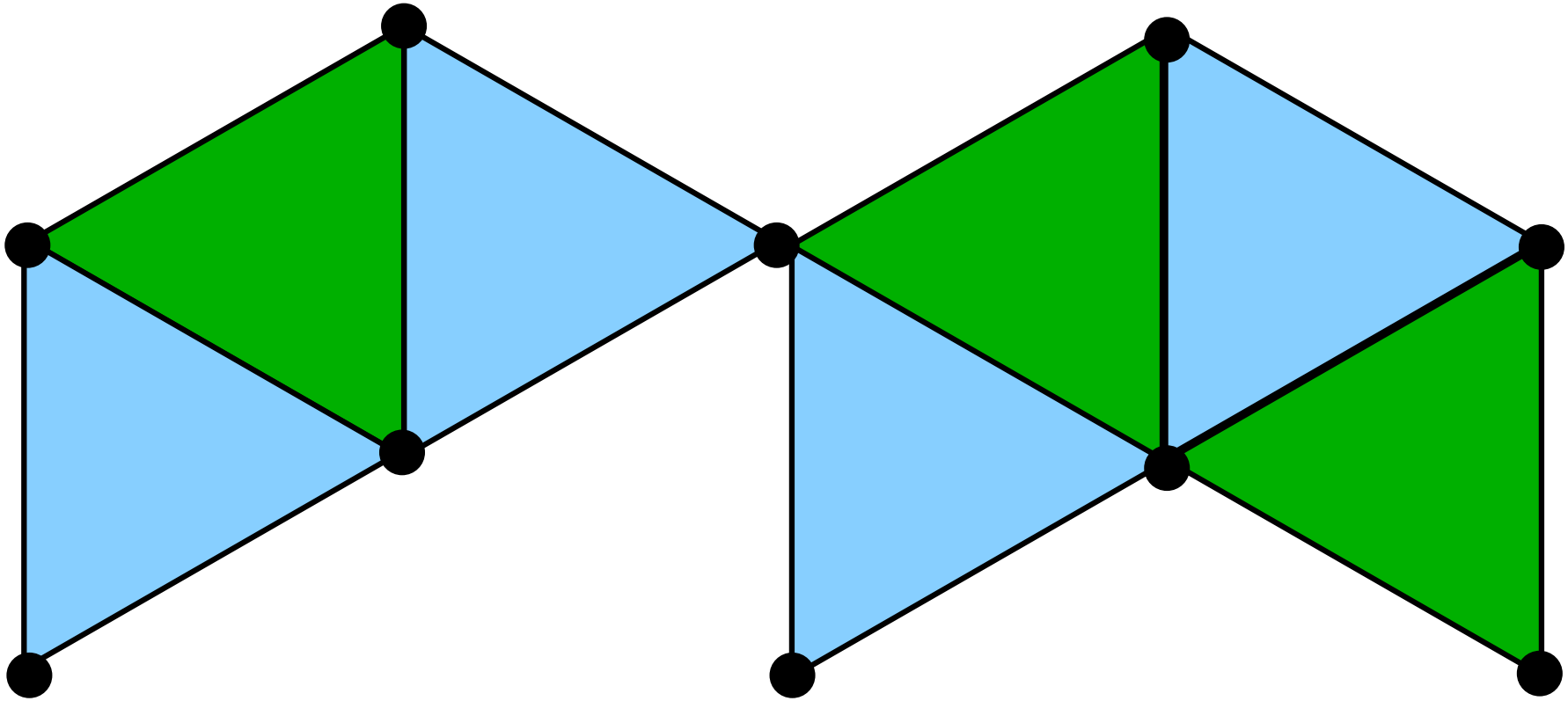

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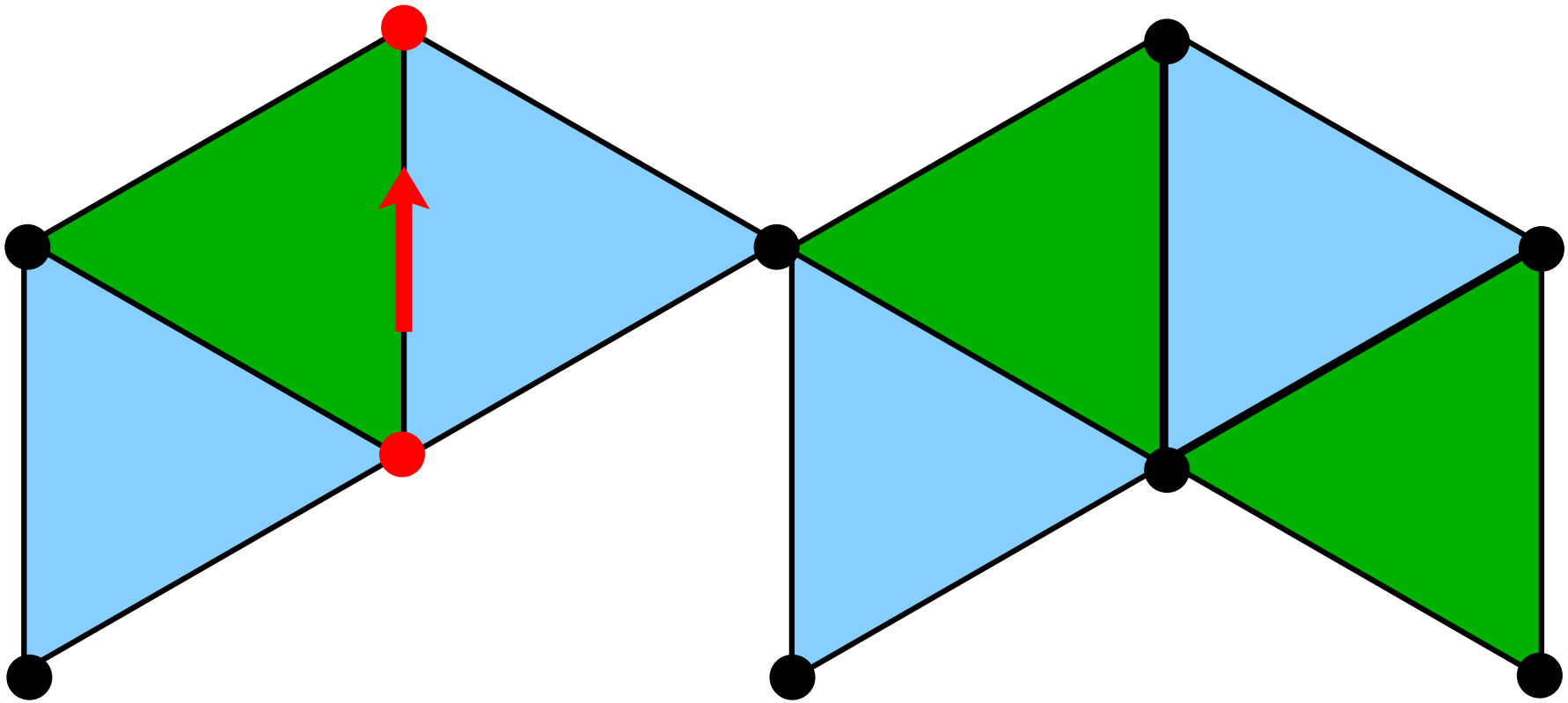

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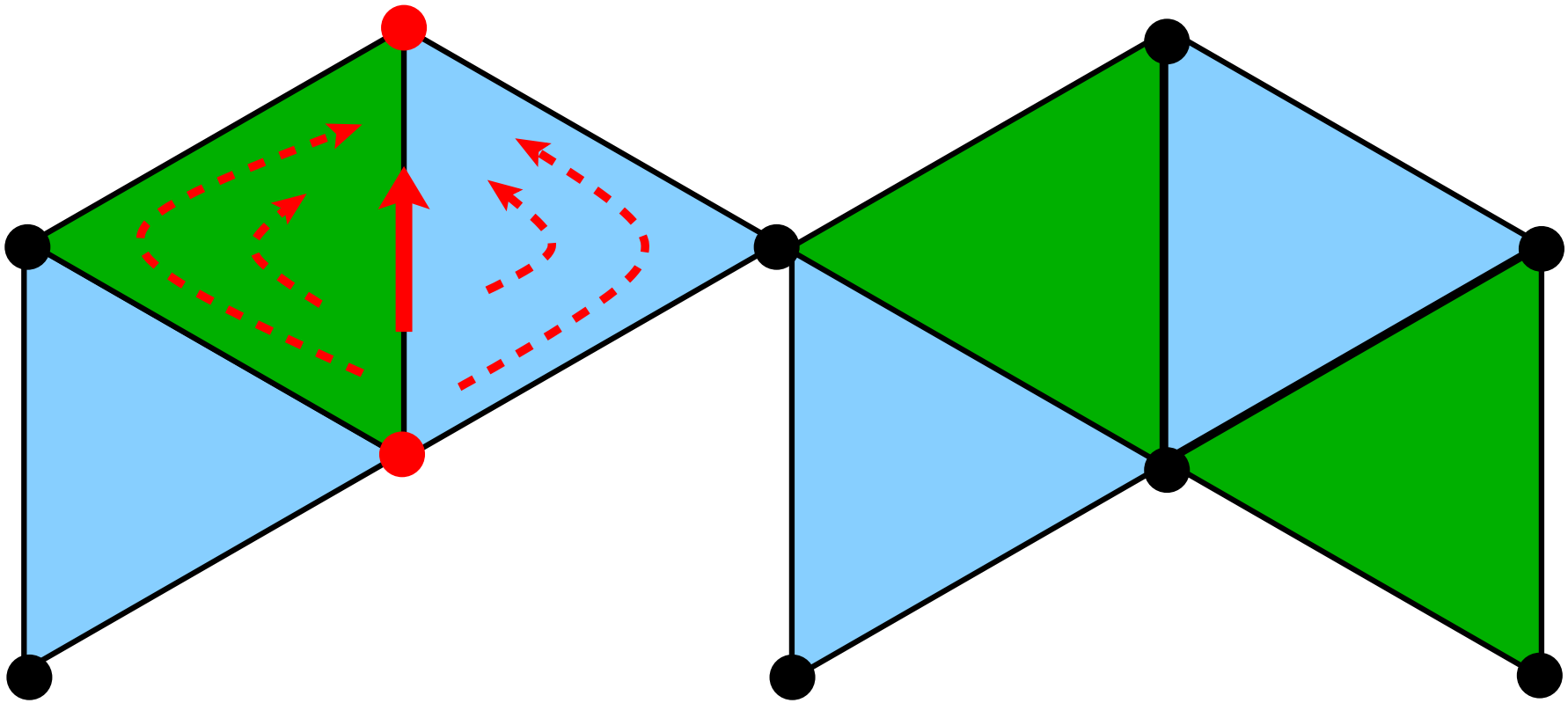

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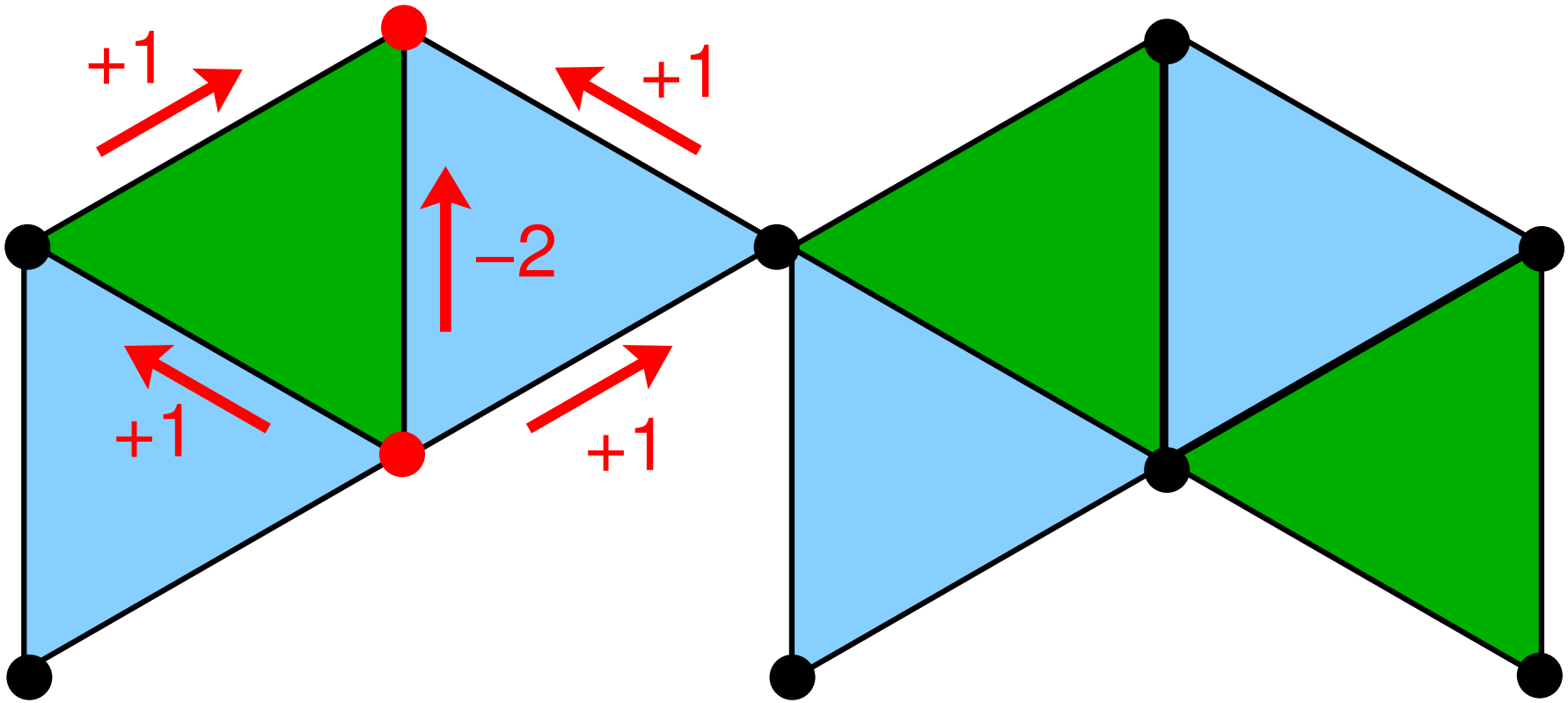

Higher-Dimensional Flow Models

What we[1] know:

- Charge conservation at vertices $\cong$ conservation of dollars
- Number of long-term behaviors = network complexity

What we don't know:

- No net conservation of current $\Longrightarrow$ very hard to characterize stability
- Some configurations are more equal than others

[1] "We" = Art Duval, Caroline Klivans, JLM