Math 725, Spring 2006
Problem Set #5
Due Friday, April 7, in class

#1. [West 3.3.7] For each $k > 1$, construct a k-regular simple graph having no perfect matching.

#2. [West 3.3.22] Let G be an X,Y-bigraph. Let H be the graph obtained from G by adding one vertex to Y if $n(G)$ is odd, then adding edges to make Y into a clique.

(a) Prove that G has a matching of size $|X|$ if and only if H has a perfect matching.

(b) Prove that if G satisfies Hall's condition (that is, $|N(S)| \geq |S|$ for all $S \subseteq X$), then H satisfies Tutte's condition (that is, $o(H - T) \leq |T|$ for all $T \subseteq V(H)$).

(c) Use parts (a) and (b) to conclude that Tutte's 1-Factor Theorem 3.3.3 implies Hall's Theorem 3.1.11.

#3. [West 4.1.9] For each choice of integers k, ℓ, m with $0 < k \leq \ell \leq m$, construct a simple graph G such that $\kappa(G) = k$, $\kappa'(G) = \ell$, and $\delta(G) = m$.

#4. [West 4.1.14] Let G be a connected graph such that for every edge e, there are cycles C_1, C_2 such that $E(C_1) \cap E(C_2) = \{e\}$. Prove that G is 3-edge-connected.

#5. [West 4.2.23] Let G be an X,Y-bigraph. Let H be the graph obtained from G by adding two new vertices s,t, an edge sx for every $x \in X$, and an edge ty for every $y \in Y$.

(a) Prove that $\alpha'(G) = \lambda_H(s,t)$.

(b) Prove that $\beta(G) = \kappa_H(s,t)$.

(So the vertex version of Menger's Theorem implies the König-Egerváry Theorem.)

#6. [West 4.2.12] Use Menger's Theorem to give a proof that $\kappa(G) = \kappa'(G)$ when G is 3-regular.