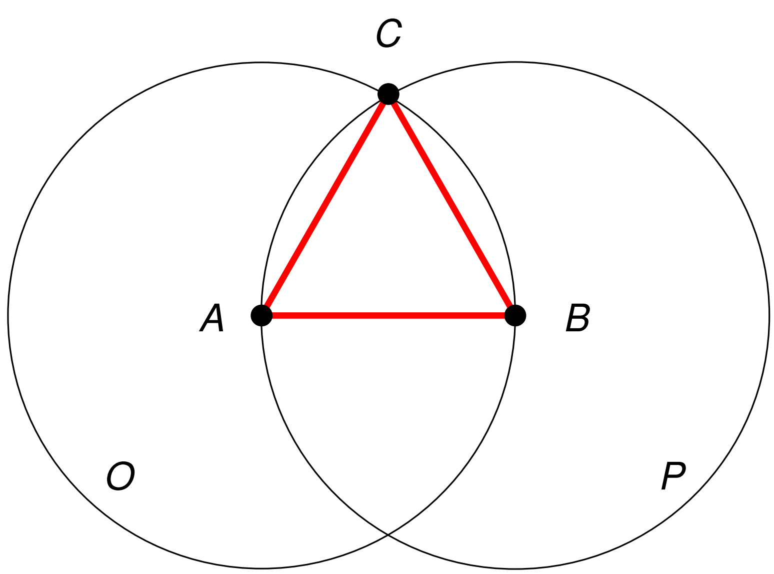
Math 409, Spring 2009
Solutions to HW #1

Note: There are correct constructions and explanations other than the ones I give here. The main purpose of this solution set is to give you an idea of how to write comprehensive explanations.

EG3: Construct an equilateral triangle ΔABC.

1. Draw two points A and B.
2. Construct a circle O centered at A and passing through B.
3. Construct a circle P centered at B and passing through A.
4. Let C be one of the points where O and P meet.
5. The triangle ΔABC will then be equilateral.

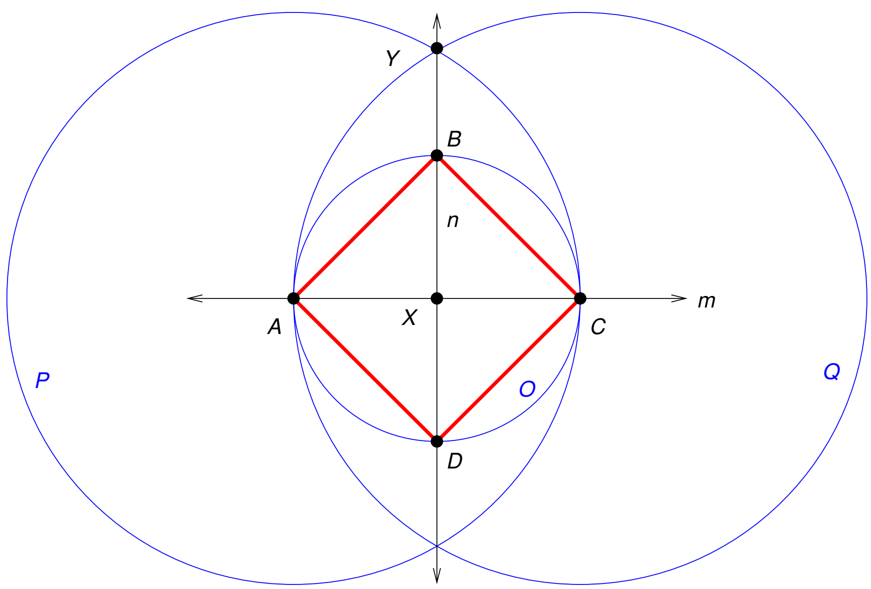


Here's why the construction works.

Since circle O is centered at A and contains both B and C (by steps 2 and 4), it follows (from the definition of a circle) that $AB = AC$. Likewise, since circle P is centered at B and contains both A and C (by steps 3 and 4), it follows (from the definition of a circle) that $AB = BC$. Therefore, $AB = AC = BC$, which means that ΔABC is an equilateral triangle.

EG9: Construct a square with vertices A, B, C, D.

1. Draw a point X, a line m containing X, and a circle O centered at X.
2. Let A and C be the two points where m meets O.
3. Construct a circle P centered at A passing through C.
4. Construct a circle Q centered at C passing through A.
5. Find one of the points where P and Q intersect; call it Y. Draw a line n through Y and X.
6. Let B and D be the two points where line n meets circle O.
7. The quadrilateral $S = ABCD$ will then be a square.



Here's why this works.

We have constructed n to be perpendicular to m at X. That is

$$\angle AXB \;=\; \angle BXC \;=\; \angle CXD \;=\; \angle DXA = 90^\circ.$$

Moreover, the segments XA, XB, XC, XD are all radii of circle O (by steps 2 and 6 of the construction), so they have equal length. Therefore, by the Side-Angle-Side rule, the triangles

$$\Delta XAB, \quad \Delta XBC, \quad \Delta XCD, \quad \Delta XDA$$

are mutually congruent. Therefore,

$$AB \;=\; BC \;=\; CD \;=\; DA.$$

We now know that S is a rhombus. We still need to explain why S is a square.

By the observation made in SA4: $\overline{AC}$ is a diameter of O and B lies on O, so $m\angle ABC = 90^\circ$. The same reasoning applies to each of the other three angles of S.

We now know that S is a quadrilateral with four equal sides and four right angles; that is, it is a square.