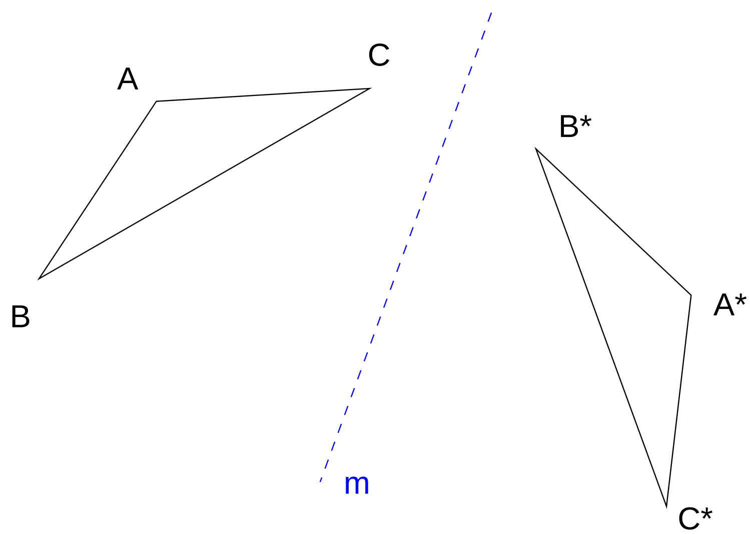
Suppose that $\Delta ABC = \Delta A^*B^*C^*$, and that you can't get one triangle from the other by fewer than two reflections. How can you tell whether two or three reflections are required?

The answer actually doesn't have to do with whether the triangles are themselves symmetric (e.g., equilateral or isosceles). Rather, what matters is the clockwise order of the vertices: is it A,B,C (which is the same thing as B,C,A or C,B,A) or A,C,B (which is the same thing as B,A,C or C,A,B)? Any reflection will reverse this clockwise order. Therefore, a composition of an even number of reflections will keep this order the same, while a composition of an odd number of reflections will reverse it.

Now, it is true that if T and T^* are equilateral or isosceles triangles and $T \cong T^*$, then there exists a line ℓ such that $r_\ell[T] = T^*$. (That is, you can get from T to T^* by a single reflection.) However — this is a subtle point — you can't control which vertex goes to which one. For example, if $T = \Delta ABC$ and $T^* = \Delta A^*B^*C^*$ are the following pair of congruent triangles, then by the "clockwiseness" agument above, there can be no reflection r_ℓ such that $r_\ell(A) = A^*$, $r_\ell(B) = B^*$, and $r_\ell(C) = C^*$. There does, however, exist a reflection r_m (shown) such that $r_\ell(A) = A^*$, $r_\ell(B) = C^*$, and $r_\ell(C) = B^*$.



On the other hand, two congruent triangles don't need to be isosceles (or equilateral) in order for one to be transformable to the other by a single reflection. After all, you can take any old scalene triangle T and any old line ℓ, and define $T^* = r_\ell[T]$; then T and T^* are congruent scalene triangles related by one reflection.