

# APPENDIX B

## Summary of Propositions

- 1.1.1 Spherical geodesics
- 1.1.2 Spherical trigonometry
- 1.1.5 Angles of a spherical triangle
- 1.1.6 Area of a spherical triangle
- 1.2.1 Hyperbolic geodesics
- 1.2.2 Hyperbolic trigonometry
- 1.2.5 Angles of a hyperbolic triangle
- 1.2.7 Area of a hyperbolic triangle
- 1.3.1 Betweenness in taxicab geometry
  
- 2.3.1 Equilateral triangles
- 2.3.2 Copy a segment
- 2.3.3 Lay off a segment
- 2.3.4 SAS
- 2.3.5 Equal sides  $\Rightarrow$  equal angles
- 2.3.6 Equal angles  $\Rightarrow$  equal sides
- 2.3.7 Lemma for 2.3.8
- 2.3.8 SSS
- 2.3.9 Bisection of angle
- 2.3.10 Bisection of segment
- 2.3.11 Raise perpendicular
- 2.3.12-14 Perpendicular bisector
- 2.3.15 Drop perpendicular
- 2.3.17 Supplementary angles equal  $180^\circ$
- 2.3.18  $180^\circ \Rightarrow$  supplementary angles
- 2.3.19 Vertically opposite angles
- 2.3.20 Exterior angle  $>$  opposite interior angle
- 2.3.21 Sum of two angles in triangle  $< 180^\circ$
- 2.3.22  $a > b \Rightarrow \alpha > \beta$
- 2.3.23  $\alpha > \beta \Rightarrow a > b$

## SUMMARY OF PROPOSITIONS

- 2.3.24 The perpendicular is the shortest line
- 2.3.25  $a + b > c$
- 2.3.26 Inside line is shorter
- 2.3.27  $a + b > c \Rightarrow$  triangle exists
- 2.3.28 Copy an angle
- 2.3.29 ASA
- 2.3.30 AAS
- 2.3.31-33 Angle bisector
- 2.3.34 Equal alternate angles  $\Rightarrow$  parallel lies
- 2.3.35 Supplementary interior, equal corresponding angles  $\Rightarrow$  parallel
  
- 3.1.1 Parallel lines  $\Rightarrow$  equal or supplementary angles
- 3.1.2 Equidistant straight lines
- 3.1.3 Parallelism is transitive
  
- 3.1.4 Construction of parallel lines
- 3.1.5 Uniqueness of parallel lines
- 3.1.6 Exterior angle equals sum of interior angles
- 3.1.7 Opposite and equal sides  $\Rightarrow$  parallelogram
- 3.1.8 Parallelograms have equal sides & angles
- 3.2.1 Area of rectangle =  $ab$
- 3.2.2 Area of parallelogram =  $bh$
- 3.2.3-4 Equal parallelograms
- 3.2.5 Area of triangle =  $bh/2$
- 3.2.6-7 Equal triangles
- 3.2.8-9 Equal triangles and bases  $\Rightarrow$  equal altitudes
- 3.2.10 Parallelogram = double of triangle
- 3.2.11 Area of spherical lune
- 3.2.12 Area of spherical triangle
- 3.2.13 Convert triangle to parallelogram
- 3.2.14 Convert parallelogram to parallelogram
- 3.2.15 Convert triangle to parallelogram
- 3.2.16 Convert polygon to parallelogram
- 3.3.1 Construct a square
- 3.3.2 Pythagoras
- 3.3.3 Converse of Pythagoras
- 3.3.4 Spherical Pythagoras
- 3.3.5 Hyperbolic Pythagoras
- 3.4.1 Golden Ratio
- 3.4.2-3 Law of Cosines
- 3.4.4 Convert rectangle to square
- 3.5.1 Distributive Law
- 3.5.2-5 Laws of Proportions
- 3.5.6 Parallel lines  $\Leftrightarrow$  proportional cuts
- 3.5.7 AAA similarity
- 3.5.8 SSS similarity
- 3.5.9 SAS similarity
  
- 4.1.1 Equal chords  $\Leftrightarrow$  equal arcs  $\Leftrightarrow$  equal angles
- 4.1.2 Semicircles are equal
- 4.1.3 Radius bisects chord
- 4.1.4 Tangent to circle
- 4.1.5 Proportional arcs and angles
- 4.2.1 Central angle equals twice the angle at the circumference
- 4.2.2 Angles at the circumference are equal
- 4.2.3 Angle on diameter equals right angle

## SUMMARY OF PROPOSITIONS

- 4.2.4 Tangent and chord
- 4.2.5 Tangent and secant
- 4.2.6 Angles in cyclic quadrilateral
- 4.2.7 Perpendicular bisectors of  $\Delta$  are concurrent
- 4.2.8 Circumscribed circle
- 4.2.9 Angle bisectors of  $\Delta$  are concurrent
- 4.2.10 Inscribed circle
- 4.3.1 Regular hexagon
- 4.3.2-3  $36^\circ, 72^\circ, 72^\circ$  triangle
- 4.3.4 Regular pentagon
- 4.3.5 Regular polygons
- 4.4.1-6 Circumference and area of circle
- 4.4.7-8 Estimating  $\pi$
  
- 5.1.1-4 Construction and uniqueness of division
- 5.2.1 Theorem of Menelaus
- 5.2.2 Theorem of Ceva
- 5.2.3 Theorem of Pappus
- 5.2.4 Theorem of Desargues
- 5.2.5 Theorem of Pascal
- 5.3.1-2 Projective points and lines
- 5.3.3-4 Theorem of Menelaus in the projective plane
  
- 6.1.1 The composition of translations is a translation
- 6.1.2 Rigid motions preserve straight lines
- 6.1.3 Agreement at two points  $\Rightarrow$  agreement on their line
- 6.1.4 Agreement at three noncollinear points  $\Rightarrow$  agreement on plane
- 6.1.5 Fixing three noncollinear points  $\Rightarrow$  identity
- 6.2.1-2 Composition of reflections
- 6.2.3 Composition of rotations
- 6.2.4 Composition of translations with rotations
- 6.3.1,3,5 Composition of glide-reflections
- 6.4.1-2 Every rigid motion is the composition of at most three reflections
- 6.4.3 The rigid motions are translations, rotations or glide-reflections
- 6.6.1 The classification of the frieze patterns
- 6.7.1 There are exactly 17 wallpaper symmetry groups
  
- 7.1.1 The effect of inversions on straight lines and circles
- 7.1.5 Inversions preserve angles
- 7.2.3 Ptolemy's theorem
- 7.3.1 Hyperbolic rigid motions
  
- 8.1.1 Euler's equation for polyhedra:  $v - e + f = 2$
- 8.2.1 The composition of rotations with intersecting axes is a rotation
- 8.2.2 The rotation group of the sphere
- 8.2.3 The rotation group of the tetrahedron
- 8.2.4 The rotation group of the octahedron