Math 243, Fall 2015
Extra Honors Problem for Week 3

Prove that every linear function $f : \mathbb{R}^n \to \mathbb{R}$ is continuous, using the precise $\varepsilon - \delta$ definition of limit (Colley, p. 100, Defn. 2.2). That is, suppose that f is given by

$$f(x_1, \ldots, x_n) = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n + d$$

where $c_1, \ldots, c_n, d \in \mathbb{R}$ are arbitrary constants. Prove that if $\mathbf{a} = (a_1, \ldots, a_n)$ is any point in $\mathbb{R}^n$, then

$$\lim_{\mathbf{x} \to \mathbf{a}} f(\mathbf{x}) = f(\mathbf{a}).$$