Math 223, Spring 2007
Review Information for Final Exam

General Information

Time, date and place of final exam: **Friday, May 18, 10:30 AM – 1:00 PM, Snow 301**.
We will start right at 10:30, so plan to arrive 10 minutes early.

Bring pens or pencils and a calculator. Check the batteries beforehand — two students may not share a calculator, and I will not have extras available.

The exam is closed-book and closed-notes; however, a list of formulas will be provided to you (see next page).

You don't need to bring a bluebook or scratch paper.

You are responsible for the following topics in addition to those covered on the two midterm tests:

- Triple integrals over arbitrary regions (pp. 312–320)
- Change of variables in double and triple integrals (pp. 322–341)
- Line integrals: definition, evaluation, and the effects of reparametrization (pp. 363–375)
- Green's Theorem: usage, application to finding areas, and the ideas behind its proof (pp. 381–388)
- Conservative vector fields, path-independence, finding potential functions, and using them to evaluate line integrals (pp. 390–397)
- Parametrized surfaces, coordinate curves, tangent and normal vectors, tangent planes, surface area (pp. 405–417)
- Surface integrals and what "orientable" means (pp. 419–435)

Time, date and place of review sessions:

- Tuesday 5/15, 1:00–2:30 PM, Snow 306. ("If you were writing a Math 223 final exam, what would you put on it?")
- Wednesday 5/16, 12:30–2:00 PM, Snow 306. (Q&A session: you bring the Q's, I'll supply the A's.)

I will hold office hours on Thursday 5/17 from 1:00–4:30 PM (in my office, Snow 541). I am also available by appointment (send e-mail to `jmartin@math.ku.edu`).

Formulas that will be given to you on the exam

Projection formula: $$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\right)\mathbf{a}$$

Quadratic approximation: $$p_2(\mathbf{x}) = f(\mathbf{a}) + Df(\mathbf{a})(\mathbf{x}-\mathbf{a}) + \mathbf{x}^T Hf(\mathbf{a})\mathbf{x}$$

Conversion between rectangular and spherical coordinates:

$$\begin{aligned} x &= \rho\sin\phi\cos\theta & \rho^2 &= x^2+y^2+z^2 \\ y &= \rho\sin\phi\sin\theta & \tan\phi &= \sqrt{x^2+y^2}/z \\ z &= \rho\cos\phi & \tan\theta &= y/x \end{aligned}$$

Change of variables for double and triple integrals:

$$\begin{aligned} \iint_D f(x,y)\,dx\,dy &= \iint_{D^*} f\big(x(u,v),y(u,v)\big)\,\frac{\partial(x,y)}{\partial(u,v)}\,du\,dv \\ \iiint_D f(x,y,z)\,dx\,dy\,dz &= \iiint_{D^*} f\big(x(u,v,w),y(u,v,w),z(u,v,w)\big)\,\frac{\partial(x,y,z)}{\partial(u,v,w)}\,du\,dv\,dw \end{aligned}$$

Area and volume elements:

$$dA = dx\,dy = r\,dr\,d\theta \qquad dV = dz\,dy\,dz = r\,dr\,dz\,d\theta = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

Line integrals:

$$\int_{\mathbf{x}} f\,ds = \int_a^b f(\mathbf{x}(t))\,\|\mathbf{x}(t)\|\,dt \qquad \int_{\mathbf{x}} \mathbf{F}\cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{x}(t))\cdot\mathbf{x}(t)\,dt$$

Green's Theorem:

$$\oint_{\partial D} M\,dx + N\,dy = \iint_D (N_x - M_y)\,dx\,dy$$

Surface integrals:

$$\iint_{\mathbf{X}} f\,dS = \iint_D f(\mathbf{X}(s,t))\,\|\mathbf{N}(s,t)\|\,ds\,dt \qquad \iint_{\mathbf{X}} \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{X}(s,t))\cdot\mathbf{N}(s,t)\,ds\,dt$$

Surface area:

$$\iint_D \sqrt{\left(\frac{\partial(x,y)}{\partial(s,t)}\right)^2 + \left(\frac{\partial(x,z)}{\partial(s,t)}\right)^2 + \left(\frac{\partial(y,z)}{\partial(s,t)}\right)^2}\,ds\,dt \quad \text{or} \quad \iint_D \sqrt{f_x^2+f_y^2+1}\,dx\,dy$$

Two useful integrals:

$$\int \sin^2 u\,du = \frac{u-\sin u\cos u}{2} + C \qquad \int \cos^2 u\,du = \frac{u+\sin u\cos u}{2} + C$$

Review Exercises

For each topic, I've listed some exercises from the textbook that might resemble exam problems. Some of these exercises have already appeared in homework assignments. **The problems on the final exam will not necessarily look like these review exercises!** You should not necessarily do all the problems listed (which you won't have time for anyway). One strategy is to pick one topic at a time and work on the review exercises until they start to become boring (which is a sign that you are more familiar with the topic in question).

Chapter 1:

- Find a parametric description of a line in $\mathbb{R}^3$, given two points on it, or a point and a direction vector, or two planes that both contain the line, etc. (§1.2: #13–20)
- Find the distance between a point and a line, or between two skew lines in $\mathbb{R}^3$, or between a point and a plane in $\mathbb{R}^3$, etc. (§1.5: #20–28)
- Prove simple vector identities involving dot and/or cross products, projections, and norms: (§1.3: #17–19; §1.4: #20; §1.6: #9–12)

Chapter 2:

- Sketch the graph and/or level curves of a function $f : X \subseteq \mathbb{R}^2 \to \mathbb{R}$ (§2.1: #10–19), or the level surfaces of a function $f : X \subseteq \mathbb{R}^3 \to \mathbb{R}$ (§2.1: #28–32)
- Evaluate a limit of a multivariable function, or prove that it does not exist (§2.2: #7–22, 28–33)
- Understand what it means for a function to be continuous at a point in its domain (§2.2: #34–42)
- Calculate the derivative matrix of a function (§2.3: #20–25)
- Find the tangent plane to the graph of a function $f : X \subseteq \mathbb{R}^2 \to \mathbb{R}$ (§2.2: #29–33) or to a level surface of a function $f : X \subseteq \mathbb{R}^3 \to \mathbb{R}$ (§2.6: #16–23)
- Understand what it means for a function to be of class C^k, and what that implies about its partial derivatives (§2.4: #9–18)
- Apply the Chain Rule in its matrix form (§2.5: #15–21)
- Calculate directional derivatives and the gradient, and determine the direction of greatest increase or decrease of a function (§2.6: #2–15)

Chapter 3:

- Sketch a parametrized curve in $\mathbb{R}^2$ or $\mathbb{R}^3$ (§3.1: #1–6)
- Determine the tangent vector (or tangent line), acceleration vector, and speed of a parametrized curve at a point (§3.1: #7–10, 15–19)
- Set up an integral for the arclength of a parametrized curve, and evaluate it if possible (§3.2: #1–10)
- Sketch a vector field in $\mathbb{R}^2$ or $\mathbb{R}^3$ (§3.3: #1–12)
- Given a vector field $\mathbf{F}$, verify that a given parametrized curve is a flow line of $\mathbf{F}$ (§3.3: #17–19) or set up (and if possible solve) a system of differential equations to find flow lines of $\mathbf{F}$ (§3.3: #20–22)
- Calculate the divergence and curl of a vector field (§3.4: #1–11)
- Prove identities involving divergence and curl (§3.4: Theorem 4.3 and Theorem 4.4 (p. 218), and #14,15,21–24)

Chapter 4:

- Calculate the linear or quadratic approximation to a function at a point, using the Hessian formula for the quadratic approximation (§4.1: #8–18)
- Find the critical points of a function, and classify them using the Hessian test (§4.2: #3–20)
- Use Lagrange multipliers to solve constrained optimization problems (§4.3: #2–8, 17–24)

Chapter 5:

- Find the area of a region in $\mathbb{R}^2$, or the volume of a region in $\mathbb{R}^3$, using a double or triple integral (§5.1: #7–15; §5.2: #21–27; §5.4: #8–10)
- Set up and evaluate the double integral of a function over a general region in $\mathbb{R}^2$ (§5.2: #2–9, 11–16)
- Set up and evaluate the triple integral of a function over a general region in $\mathbb{R}^3$ (§5.4: #11–18)
- Change the order of integration in a double integral (§5.3: #1–11) or a triple integral (§5.4: #21–24)
- Find a linear transformation that maps any given parallelogram to the unit square in $\mathbb{R}^2$ (§5.5: #1–6)
- Apply the change-of-variables formula to evaluate a double or triple integral (§5.5:#9–12)
- Convert double integrals between rectangular and polar coordinates (§5.5: #13–18, 20–23)
- Convert triple integrals between rectangular, cylindrical, and spherical coordinates (§5.5: #24–32)

Chapter 6:

- Set up and evaluate a scalar or vector line integral of a function $f : \mathbb{R}^2 \to \mathbb{R}$ along a parametrized curve in $\mathbb{R}^2$ (§6.1: #1–11, 13, 14, 17–21)
- Understand what happens to a line integral when the curve is reparametrized (in particular, know what is meant by an orientation of a curve) (§6.1: #15)
- Use Green's Theorem to convert a line integral to a double integral, or vice versa (§6.2: #5–8,11)
- Use Green's Theorem to find the area of a bounded region (§6.2: #9,10,12–16)
- Use Green's Theorem to deduce facts about line integrals of conservative vector fields (§6.2: #19–21, §6.3: #1,2)
- Determine whether a vector field is conservative, and if so find a scalar potential function (§6.3: #3–16)
- Use the scalar potential function of a conservative vector field to evaluate line integrals (§6.3: #17–24)

Chapter 7:

- For a given parametrized surface $\mathbf{X} : \mathbb{R}^2 \to \mathbb{R}^3$, find coordinate curves, tangent vectors, normal vectors and tangent planes, and express the underlying surface as the graph of a function $f : \mathbb{R}^2 \to \mathbb{R}$ or as a level surface of a function $f : \mathbb{R}^3 \to \mathbb{R}$ (§7.1: #1–4,6–11,14–17)
- Find the surface area of a parametrized surface (§7.1: #18–21) or of the graph of a function $f : \mathbb{R}^2 \to \mathbb{R}$ (§7.1: #22–26)
- Set up and evaluate a scalar or vector surface integral of a function $f : \mathbb{R}^3 \to \mathbb{R}$ along a parametrized surface in $\mathbb{R}^3$ (§7.2: #1,2,5,6,10–18)
- Know what happens to a surface integral when the surface is reparametrized (§7.2: #4)