Math 141 Honors Problems #3
Due date: Tuesday, 9/8/09

HP5 [4 points] Define a function S with domain $\mathbb{R}$ as follows: $S(x)$ is the number obtained by writing x as a decimal and swapping the first two digits after the decimal point. For example:

$$\begin{aligned} S(0) &= 0 \\ S(0.12) &= 0.21 \\ S(-0.12) &= -0.21 \\ S(0.12345) &= 0.21345 \\ S(0.11111) &= 0.11111 \\ S(\pi) &= 3.4115926535\cdots \end{aligned}$$

For which real numbers a does $\lim_{x\to a^+} S(x)$ exist? (Suggestion: Start by choosing a few random values for a and working out the limit for the values you've chosen. Then try to determine a general pattern.)

If $\lim_{x\to a^+} S(x)$ exists, must it equal $S(a)$?

Answer the same questions for $\lim_{x\to a^-} S(x)$ and $\lim_{x\to a} S(x)$.

For which real numbers a is S continuous at a?

HP6 [3 points] Suppose that $p(x)$ and $q(x)$ are any two polynomials: that is,

$$\begin{aligned} p(x) &= a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0, \\ q(x) &= b_m x^m + b_{m-1}x^{m-1} + \cdots + b_2x^2 + b_1x + b_0, \end{aligned}$$

where n and m are nonnegative integers and $a_n, \ldots, a_0, b_m, \ldots, b_0$ are real numbers. This problem is about the limit

$$\lim_{x\to 0} \frac{p(x)}{q(x)}.$$

a. Under what conditions on $n, m, a_n, \ldots, a_0, b_m, \ldots, b_0$ does the limit equal 0?

b. Under what conditions does the limit equal a nonzero real number? What nonzero real number is it?

c. Under what conditions does the limit not exist?

Your answers should be in terms of the coefficients $a_n, \ldots, a_0, b_m, \ldots, b_0$, and should include a complete explanation. (For instance, it is not sufficient to only give an example of each of the three cases.)

(Note: Don't assume that the rational function $p(x)/q(x)$ is in lowest terms. The answer I'm looking for applies to every possible pair of polynomials, even ones that that have common factors.)