#1 [4 pts] Evaluate

$$\frac{d}{dx}\left[\int_x^0 r^r\,dr\right].$$

By Part I of the Fundamental Theorem of Calculus,

$$\frac{d}{dx}\left[\int_x^0 r^r\,dr\right] = \frac{d}{dx}\left[-\int_0^x r^r\,dr\right] = \boxed{-x^x.}$$

#2 [5 pts] Find all values of a such that

$$\int_a^{2a}(6s^2-7)\,ds = 0.$$

First, evaluate the integral in terms of a, using Part II of the Fundamental Theorem of Calculus,

$$\begin{aligned}\int_a^{2a}(6s^2-7)\,ds &= 2s^3-7s\Big]_a^{2a}\\ &= (2(2a)^3-7(2a)) - (2a^3-7a)\\ &= 16a^3-14a-2a^3+7a\\ &= 14a^3-7a = 7a(2a^2-1).\end{aligned}$$

Setting this to zero yields the answers $\boxed{a=0,\ a=\pm 1/\sqrt{2}.}$

#3 [5 pts] Evaluate $\int_0^{\pi/2}(\cos x)\,2^{\sin x}\,dx.$

Use substitution with $u=\sin x$, $du=\cos x\,dx$ to get

$$\begin{aligned}\int_0^{\pi/2}(\cos x)\,2^{\sin x}\,dx &= \int_{\sin(0)}^{\sin(\pi/2)} 2^u\,du\\ &= \int_0^1 2^u\,du\\ &= \frac{2^u}{\ln 2}\Big]_0^1 = \frac{2}{\ln 2}-\frac{1}{\ln 2} = \boxed{\frac{1}{\ln 2}.}\end{aligned}$$

#4 [6 pts] Evaluate $\int (\ln x)^2\, dx$.

Use integration by parts. This integral is $\int u\, dv$, where $u = (\ln x)^2$ and $dv = dx$. Therefore $du = \frac{2\ln x}{x}\, dx$ and $v = x$, yielding

$$\int (\ln x)^2\, dx \;=\; x(\ln x)^2 \;-\; \int 2\ln x\, dx. \tag{1}$$

We now need to find $\int \ln x\, dx$. Again, we can use integration by parts with $u = \ln x$ and $dv = dx$, so $du = dx/x$ and $v = x$. This gives

$$\int \ln x\, dx \;=\; x\ln x \;-\; \int dx \;=\; x\ln x - x + C, \tag{2}$$

and substituting (2) into (1) yields

$$\int (\ln x)^2\, dx \;=\; x(\ln x)^2 \;-\; 2(x\ln x - x + C)$$

or more simply

$$\boxed{\int (\ln x)^2\, dx \;=\; x(\ln x)^2 \;-\; 2x\ln x \;+\; 2x \;+\; C.}$$

Bonus Problem [5 honors points] Evaluate $\int \frac{e^{3x} + e^x}{e^{2x} - 1}\, dx$.

Write the integral as

$$\int \frac{(e^{2x}+1)e^x}{e^{2x}-1}\, dx$$

and make the substitution $u = e^x$, $du = e^x\, dx$ to obtain

$$\int \frac{u^2+1}{u^2-1}\, du.$$

By polynomial long division, this becomes

$$\begin{aligned} \int \left(1 + \frac{2}{u^2-1}\right) du &= \int du + \int \frac{2}{u^2-1}\, du \\ &= u + \int \frac{2}{u^2-1}\, du. \end{aligned}$$

Note that $u^2 - 1 = (u-1)(u+1)$; we therefore need to find the partial fraction decomposition

$$\frac{2}{u^2-1} \;=\; \frac{A}{u-1} + \frac{B}{u+1} \;=\; \frac{A(u+1)+B(u-1)}{u^2-1} \;=\; \frac{Au+A+Bu-B}{u^2-1} \;=\; \frac{(A+B)u+(A-B)}{u^2-1}.$$

So $A+B=0$ and $A-B=2$; this system has the solution $A=1$, $B=-1$. Therefore

$$\begin{aligned}\int \frac{2}{u^2-1}\,du &= \int \frac{du}{u-1} - \int \frac{du}{u+1} \\ &= \ln|u-1| + \ln|u+1| + C\end{aligned}$$

(by substituting $v=u-1$, $dv=du$ in the first integral and substituting $w=u+1$, $dw=du$ in the second one). Therefore

$$\int \frac{u^2+1}{u^2-1}\,du = u + \ln|u-1| + \ln|u+1| + C$$

and

$$\int \frac{(e^{2x}+1)e^x}{e^{2x}-1}\,dx = e^x + \ln|e^x-1| + \ln|e^x+1| + C.$$