For problems #1 through #4, differentiate the given functions. You don't have to simplify your answers, but if you do, the simplification must be algebraically correct.

#1. $\boldsymbol{f(t) = \dfrac{t}{e^{2t} - e^{-2t}}}$

Use the Quotient Rule, then the Chain Rule:

$$f'(t) = \frac{(e^{2t} - e^{-2t}) - t \cdot \dfrac{d}{dt}\left(e^{2t} - e^{-2t}\right)}{(e^{2t} - e^{-2t})^2}$$

$$= \frac{e^{2t} - e^{-2t} - t(2e^{2t} + 2e^{-2t})}{(e^{2t} - e^{-2t})^2}.$$

#2. $\boldsymbol{g(x) = \cos^2 x \sin x}$

Use the Product Rule, then the Chain Rule to differentiate $\cos^2 x$:

$$\begin{aligned} g'(x) &= (\cos^2 x)(\cos x) + (2\cos x)(-\sin x)(\sin x) \\ &= \cos^3 x - 2\cos x \sin^2 x. \end{aligned}$$

#3. $\boldsymbol{q(x) = \dfrac{xe^x}{\arctan x}}$

Use the Quotient and Product Rules:

$$q'(x) = \frac{(\arctan x) \cdot \dfrac{d}{dx}(xe^x) - (xe^x) \cdot \frac{d}{dx} \arctan x}{(\arctan x)^2}$$

$$= \frac{(\arctan x)(xe^x + e^x) - \dfrac{xe^x}{1 + x^2}}{(\arctan x)^2}.$$

#4. Find the tangent line to the curve defined by the equation

$$y^3 + xy + x^3 = 3x^2$$

at the point $(1, 1)$.

We need to find $y' = \frac{dy}{dx}$. It doesn't look like it's possible to solve the equation for y, but we can use implicit differentiation to find y':

$$\begin{aligned}
\frac{d}{dx}\left[y^3 + xy + x^3\right] &= \frac{d}{dx}\left[3x^2\right] \\
3y^2y' + (xy' + y) + 3x^2 &= 6x \\
3y^2y' + xy' &= 6x - 3x^2 - y \\
y' &= \boxed{\frac{6x - 3x^2 - y}{3y^2 + x}.}
\end{aligned}$$

Plugging in $x = 1, y = 1$ gives $y' = \frac{1}{2}$. This is the slope of the tangent line at $(1, 1)$, and the equation of the tangent line is therefore

$$(y - 1) = \frac{1}{2}(x - 1).$$

Bonus problem #1: Use implicit differentiation to calculate $\frac{d}{dx}$ arccos x. Express your answer as an algebraic function of x.

Differentiate both sides of the equation $\cos(\arccos x) = x$:

$$\begin{aligned}
-\sin(\arccos x) \cdot \frac{d}{dx}(\arccos x) &= 1 \\
\frac{d}{dx}(\arccos x) &= -\frac{1}{\sin(\arccos x)}.
\end{aligned}$$

This is not an algebraic function of x, but we can make it into one by using the identity $\sin^2\theta + \cos^2\theta = 1$. Plugging in $\theta = \arccos x$, this identity becomes

$$\begin{aligned}
\sin^2(\arccos x) + \cos^2(\arccos x) &= 1 \\
\sin^2(\arccos x) + x^2 &= 1 \\
\sin^2(\arccos x) &= 1 - x^2 \\
\sin(\arccos, x) &= \sqrt{1 - x^2}.
\end{aligned}$$

(This could also be discovered by drawing a right triangle with an angle θ such that $\cos\theta = x$.) Therefore,

$$\boxed{\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1 - x^2}}.}$$

Bonus problem #2: What can you deduce about the expression arccos x + arcsin x?

Since $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1 - x^2}}$, the previous equation implies that $\frac{d}{dx}(\arccos x + \arcsin x) = 0$. Therefore, $\arccos x + \arcsin x$ $\boxed{\text{must be a constant.}}$