

Problem: Calculate $\frac{d}{dx} (x^{\arcsin x})$.

The simplest way is to use logarithmic differentiation. Let

$$y = x^{\arcsin x}, \quad (1)$$

so that

$$\ln y = \ln x^{\arcsin x} = (\arcsin x)(\ln x). \quad (2)$$

We wish to find $\frac{dy}{dx}$. Differentiating both sides of (2) with respect to x yields

$$\frac{d}{dx} [\ln y] = \frac{d}{dx} [(\arcsin x)(\ln x)]. \quad (3)$$

Applying the Chain Rule to the left side of (3) and the Product Rule to the right side, we obtain

$$\frac{1}{y} \frac{dy}{dx} = (\arcsin x) \cdot \frac{1}{x} + \frac{1}{\sqrt{1-x^2}} \cdot \ln x,$$

that is,

$$\frac{dy}{dx} = y \left(\frac{\arcsin x}{x} + \frac{\ln x}{\sqrt{1-x^2}} \right). \quad (4)$$

Finally, we can substitute (1) into (4) to obtain the final answer:

$$\frac{dy}{dx} = x^{\arcsin x} \left(\frac{\arcsin x}{x} + \frac{\ln x}{\sqrt{1-x^2}} \right).$$