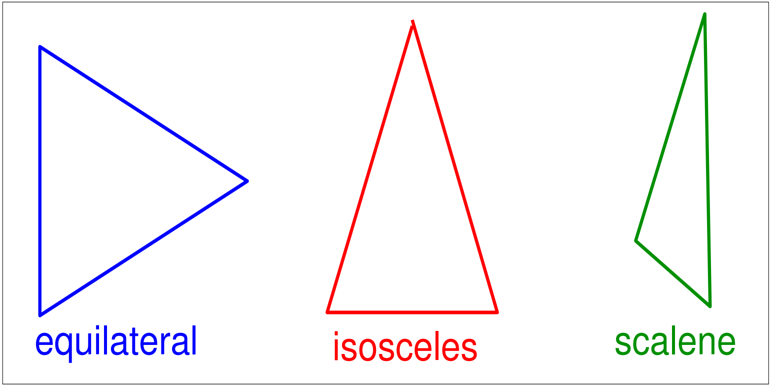
Defining Symmetry

Which of these triangles is the most symmetric?



Rigid Motions

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Rotations, reflections, translations, and glide reflections are all examples of rigid motions.

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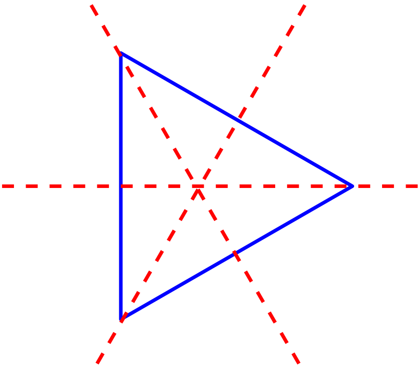
An equilateral triangle is **more symmetric** than an isosceles or scalene triangle because it has **more different rigid motions**.

Equilateral: Six symmetries

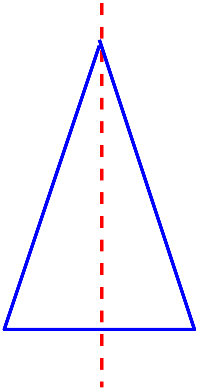


Three reflections

Three rotations

Rigid Motions

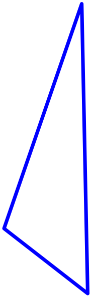
Isosceles: Two symmetries



One reflection

One rotation

Scalene: One symmetry



No reflections

One rotation

Rigid Motions

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(In other words, the more different ways you can pick up a transparency of the figure, move it around, and put it back on the original figure, the more symmetric it is.)

Every figure has at least one rigid motion,
the **identity motion** (which doesn't move it at all).

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A rigid motion is defined by **where each point ends up**, not by how it gets there.

KU

Rotate 270 ° clockwise

KU

KU

Rotate 90 ° counterclockwise

KU

Same rigid motion!

Rigid Motions

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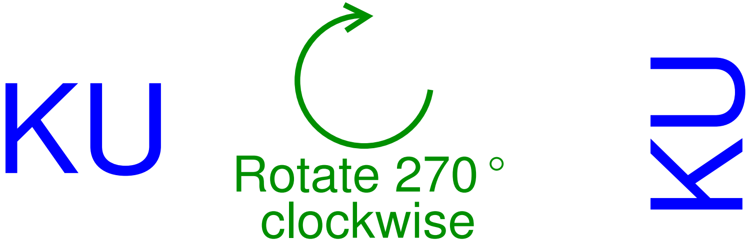
- If a motion $\mathcal{M}$ moves a point P to a point P', then P' is called the **image** of P. Notation:

$$\mathcal{M}(P) = P'.$$

- P and P' together are called a **point-image pair**.

A Point-Image Pair



A Point-Image Pair

Reflections

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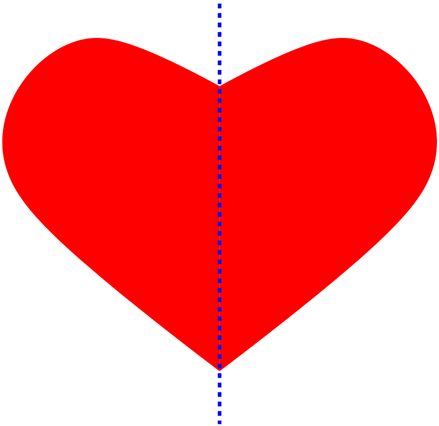

Properties of Reflections

Property 1: If $\mathcal{M}$ is a reflection and we know its axis of reflection, then we can find the image of any point under $\mathcal{M}$.

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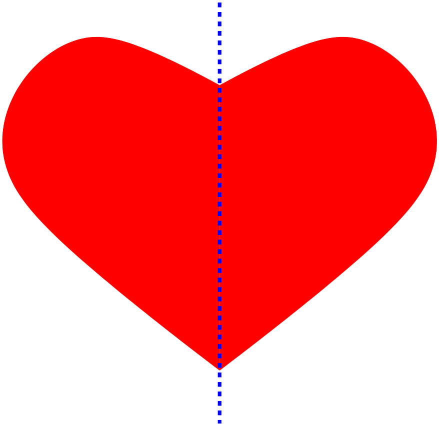

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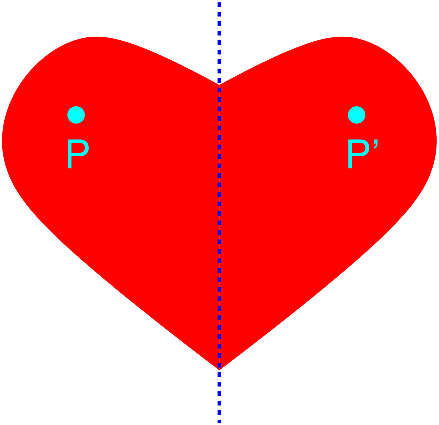

Properties of Reflections

Property 1: If $\mathcal{M}$ is a reflection and we know its axis of reflection, then we can find the image of any point under $\mathcal{M}$.

Therefore, every reflection is completely determined by its axis of reflection.

Properties of Reflections

Property 2: If $\mathcal{M}$ is a reflection, and we know a point P and its image P' (provided that $P' \neq P$), then we can find the image of any other point.

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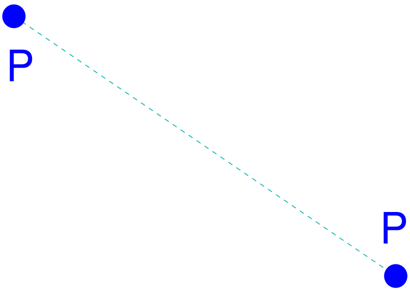

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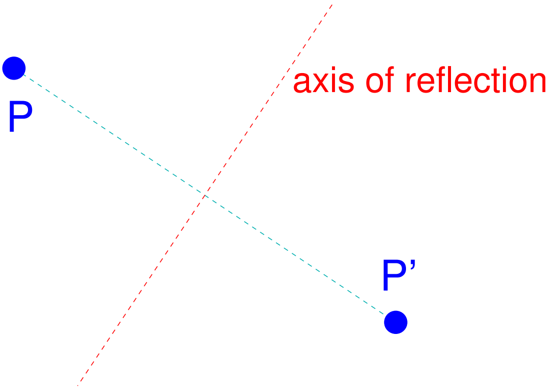

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Property 2: If $\mathcal{M}$ is a reflection, and we know a point P and its image P' (provided that $P' \neq P$), then we can find the image of any other point.

Therefore, every reflection is completely determined by one point-image pair.

Properties of Reflections

Property 3: If $\mathcal{M}$ is a reflection with axis L,
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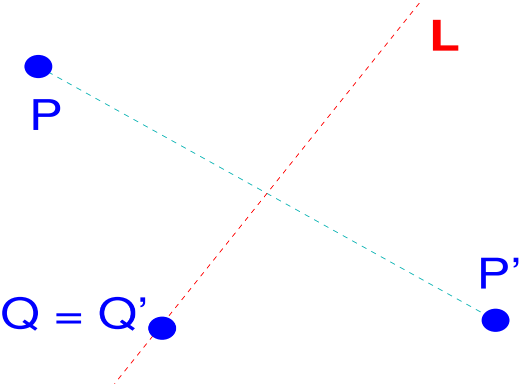

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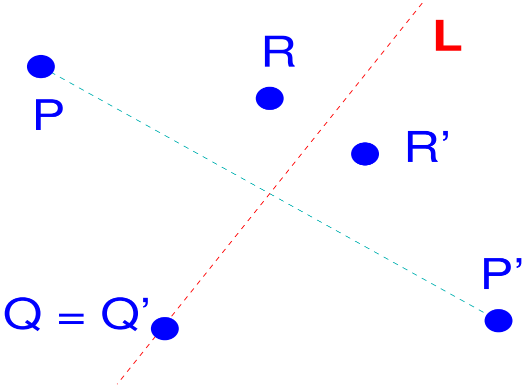

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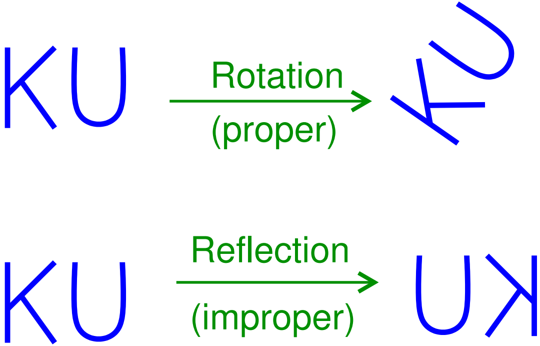

Properties of Reflections

Property 5: If P' is the image of P under a reflection, then $(P')' = P$.

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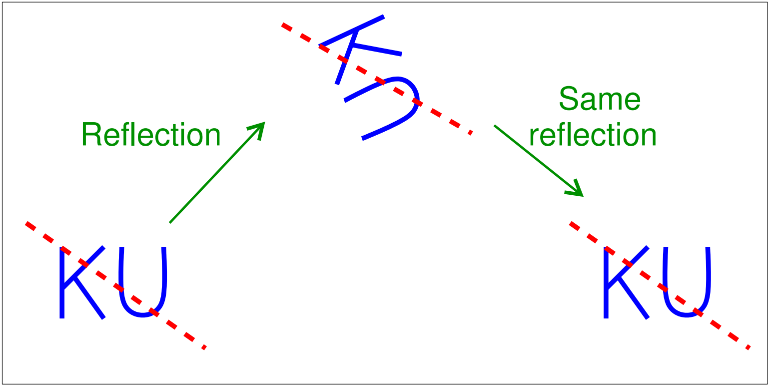

I.e., repeating a reflection twice gives the **identity motion**.

Properties of Reflections: Summary

1. Every reflection is completely determined by its axis of reflection.
2. Every reflection is completely determined by a single point-image pair P and P' (provided that $P \neq P'$).
3. The fixed points of a reflection are exactly the points on its axis of reflection.
4. Reflections are **improper** rigid motions (they reverse clockwise and counterclockwise orientations).
5. Applying the same reflection twice gives the identity motion.

Rotations

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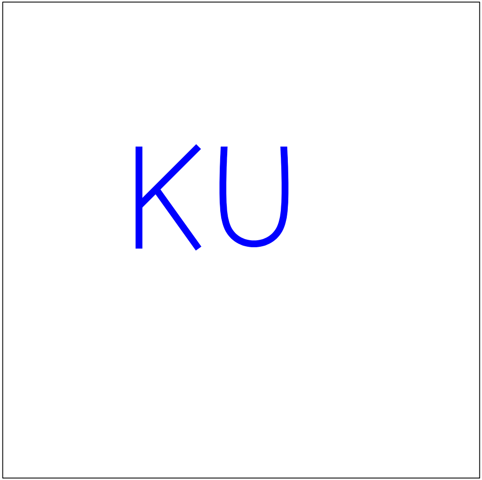

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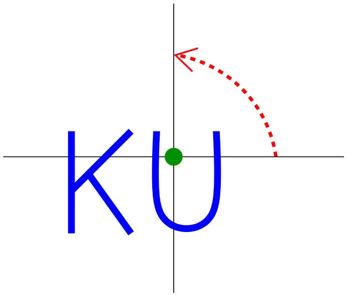
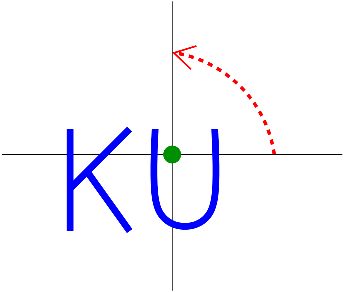


Rotation by 90° clockwise around rotocenter O

Rotations



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Rotations

Question: How many point-image pairs determine a rotation? ★

In other words, if you know that $\mathcal{M}$ is a rotation and you know that P' is the image of P, do you know what $\mathcal{M}$ is?

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How about if you know that P' is the image of P **and** that Q' is image of Q?

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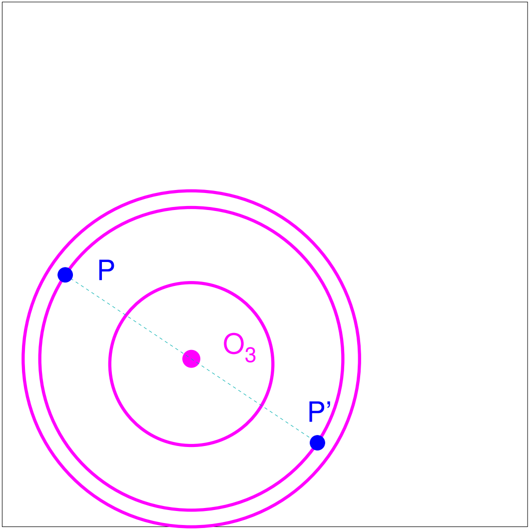
Related question: Can a rotation have any fixed point other than its rotocenter?

Rotations



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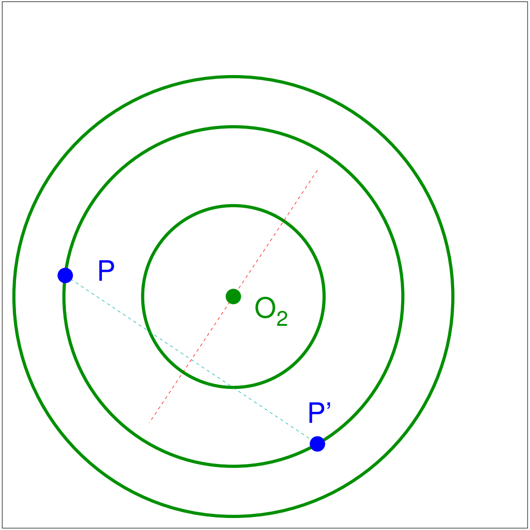

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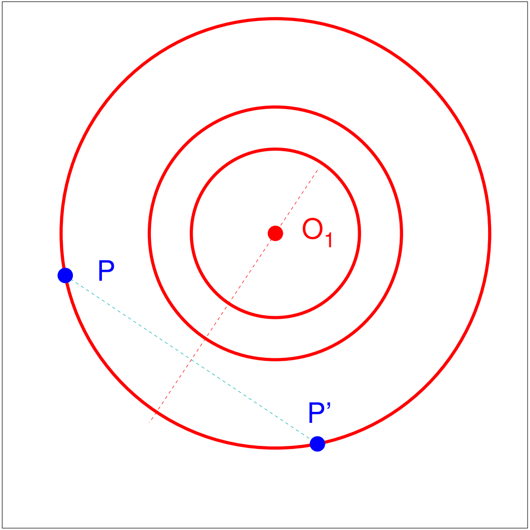

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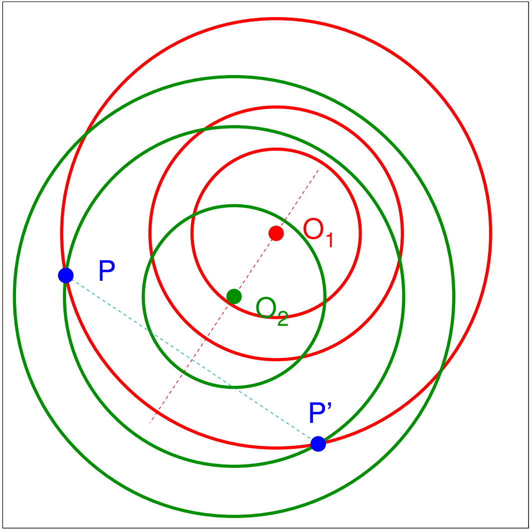

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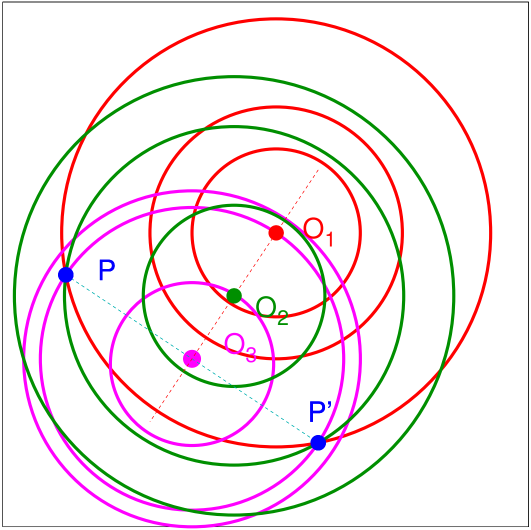

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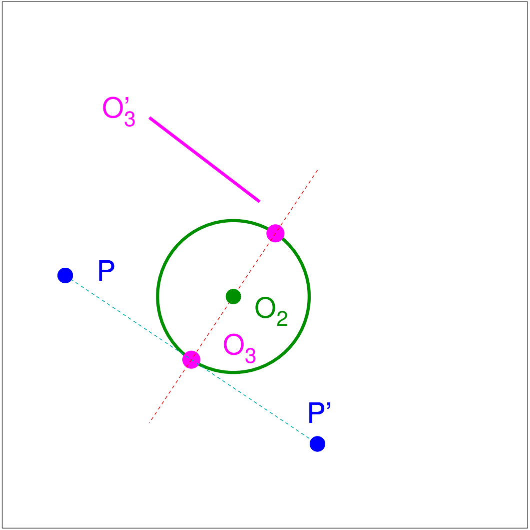

Properties of Rotations

1. A 360° rotation is equivalent to the identity motion. (So are720°, 1040°, $\dots$)
2. Rotations are **proper** rigid motions (they preserve clockwise/counterclockwise orientations). P and P' (provided that $P \neq P'$).
3. A rotation that is not the identity has only one fixed point — its rotocenter.
4. A reflection is determined by any **two** point-image pairs P, P' and Q, Q'.